Random Variables

Metal Cylinder Production

a) $P(49.8 < X < 50.1) = F(50.1) - F(49.8)$

b) $P(49.8 \le X \le 50.1) = F(50.1) - F(49.8)$

c) $P(49.8 \le X < 50.1) = F(50.1) - F(49.8)$

d) $P(49.8 < X \le 50.1) = F(50.1) - F(49.8)$

All four are **identical**!

Random Variables

Metal Cylinder Production

a) $P(49.8 < X < 50.1)$

b) $P(49.8 < X \le 50.1)$

c) $P(49.8 \le X < 50.1)$

d) $P(49.8 \le X \le 50.1)$

But be careful when using "F(b) – F(a)" technique with ***discrete*** *random variables!!! …*

Random Variables

Consider the following pmf

$$f(x) = \begin{cases} 1/36 & \text{if } x = 2 \\ 2/36 & \text{if } x = 3 \\ 3/36 & \text{if } x = 4 \\ 4/36 & \text{if } x = 5 \\ 5/36 & \text{if } x = 6 \\ 6/36 & \text{if } x = 7 \\ 5/36 & \text{if } x = 8 \\ 4/36 & \text{if } x = 9 \\ 3/36 & \text{if } x = 10 \\ 2/36 & \text{if } x = 11 \\ 1/36 & \text{if } x = 12 \end{cases}$$

Random Variables

Consider the following pmf

$$f(x) = \begin{cases} 1/36 & \text{if } x = 2 \\ 2/36 & \text{if } x = 3 \\ 3/36 & \text{if } x = 4 \\ 4/36 & \text{if } x = 5 \\ 5/36 & \text{if } x = 6 \\ 6/36 & \text{if } x = 7 \\ 5/36 & \text{if } x = 8 \\ 4/36 & \text{if } x = 9 \\ 3/36 & \text{if } x = 10 \\ 2/36 & \text{if } x = 11 \\ 1/36 & \text{if } x = 12 \end{cases}$$

Given:

a) $P(3 < X < 6)$

b) $P(3 \le X < 6)$

c) $P(3 < X \le 6)$

d) $P(3 \le X \le 6)$

Which one equals F(6) – F(3)?

Random Variables

Computer tape errors

Let the random variable X be the distance in feet between bad records on a used computer tape. Suppose that a reasonable probability model for X is given by the pdf

$$f(x) = \frac{1}{40} e^{-x/40} \quad \text{for } 0 \le x < \infty.$$

Random Variables

Computer tape errors

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$$f(x) = \frac{1}{40} e^{-x/40} \quad \text{for } 0 \le x < \infty.$$

Is this a valid pdf?

Random Variables

Computer tape errors

Let the random variable X be the distance in feet between bad records on a used computer tape. Suppose that a reasonable probability model for X is given by the pdf

$$f(x) = \frac{1}{40} e^{-x/40} \qquad \text{for } 0 \le x < \infty.$$

Find the probability that the distance between bad records is greater than 40 feet.

Random Variables

Computer tape errors

Continuing with this example, if the pdf of X is

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{40} e^{-x/40} & 0 < x < \infty \end{cases}$$

determine the cdf of X.

Random Variables

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{40} e^{-x/40} & 0 < x < \infty \end{cases}$$

Random Variables

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Remember that the cdf is defined for all real numbers.

Random Variables

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{40} e^{-x/40} & 0 < x < \infty \end{cases}$$

Remember that the cdf is defined for all real numbers.

- for $x < 0$ $F(x) = 0$

Random Variables

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{40} e^{-x/40} & 0 < x < \infty \end{cases}$$

Remember that the cdf is defined for all real numbers.

- for $x < 0$ $F(x) = 0$
- for $x > 0$ …

$F(x) =$

Random Variables

$$f(x)=\begin{cases}0 & -\infty<x<0\\ \frac{1}{40}e^{-x/40} & 0<x<\infty\end{cases}$$

Remember that the cdf is defined for all real numbers.

- for x < 0 F(x) = 0
- for x > 0 …

$$F(x)=\int_0^x f(x)dx$$

Random Variables

$$f(x)=\begin{cases}0 & -\infty<x<0\\ \frac{1}{40}e^{-x/40} & 0<x<\infty\end{cases}$$

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- for x < 0 F(x) = 0
- for x > 0 …

$$F(x)=\int_0^x f(x)dx=\int_0^x \frac{1}{40}e^{-x/40}dx$$

Random Variables

$$f(x)=\begin{cases}0 & -\infty<x<0\\ \frac{1}{40}e^{-x/40} & 0<x<\infty\end{cases}$$

Remember that the cdf is defined for all real numbers.

- for x < 0 F(x) = 0
- for x > 0 …

$$F(x)=\int_0^x f(x)dx=\int_0^x \frac{1}{40}e^{-x/40}dx=1-e^{-x/40}$$

Random Variables

$$f(x)=\begin{cases}0 & -\infty<x<0\\ \frac{1}{40}e^{-x/40} & 0<x<\infty\end{cases}$$

$$F(x)=\begin{cases}0 & x<0\\ 1-e^{-x/40} & x>0\end{cases}$$

Remember that the cdf is defined for all real numbers.

- for x < 0 F(x) = 0
- for x > 0 …

$$F(x)=\int_0^x f(x)dx=\int_0^x \frac{1}{40}e^{-x/40}dx=1-e^{-x/40}$$

Random Variables

Another example…

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

Random Variables

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

$$F(x)=\begin{cases}0 & x<0\\ y^2 & 0<x<1\\ 1 & x>1\end{cases}$$

Random Variables

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

$$F(x)=\begin{cases}0 & x<0\\ y^2 & 0<x<1\\ 1 & x>1\end{cases}$$

2a. P(½ < Y < ¾)

Random Variables

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

$$F(x)=\begin{cases}0 & x<0\\ y^2 & 0<x<1\\ 1 & x>1\end{cases}$$

2a. P(½ < Y < ¾) = $F(¾) - F(½)$ = $\frac{5}{16}$

Random Variables

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

$$F(x)=\begin{cases}0 & x<0\\ y^2 & 0<x<1\\ 1 & x>1\end{cases}$$

2b. P(¼ < Y < 2)

Random Variables

Let Y be a continuous random variable with pdf g(y) = 2y, for 0 < y < 1.

1. Find the cdf of Y.

$$F(x)=\begin{cases}0 & x<0\\ y^2 & 0<x<1\\ 1 & x>1\end{cases}$$

2b. P(¼ < Y < 2) = $F(2) - F(¼)$ = $\frac{15}{16}$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}+12x & 0<x<3\end{cases}$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}+12x & 0<x<3\end{cases}$$

- Determining if this is a valid pdf

Random Variables

Piecewise pdf's

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$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}+12x & 0<x<3\end{cases}$$

- Determining if this is a valid pdf

$$\int_{-2}^{0}\left[\frac{15}{64}+\frac{x}{64}\right]dx \;+\; \int_{0}^{3}\left[\frac{3}{8}+12x\right]dx \stackrel{?}{=} 1$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}+12x & 0<x<3\end{cases}$$

- Determining if this is a valid pdf no!

$$\int_{-2}^{0}\left[\frac{15}{64}+\frac{x}{64}\right]dx \;+\; \int_{0}^{3}\left[\frac{3}{8}+\;\;x\right]dx \neq 1$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}+cx & 0<x<3\end{cases}$$

- Determining if this is a valid pdf can redefine the pdf

$$\int_{-2}^{0}\left[\frac{15}{64}+\frac{x}{64}\right]dx \;+\; \int_{0}^{3}\left[\frac{3}{8}+\;cx\right]dx \;=\; 1$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}-\frac{x}{8} & 0<x<3\end{cases}$$

- Determining if this is a valid pdf

$$\int_{-2}^{0}\left[\frac{15}{64}+\frac{x}{64}\right]dx \;+\; \int_{0}^{3}\left[\frac{3}{8}+\;cx\right]dx \;=\; 1$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}-\frac{x}{8} & 0<x<3\end{cases}$$

- Calculating probability values

P(-2 < X < -1) =

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x)=\begin{cases}\frac{15}{64}+\frac{x}{64} & -2<x<0\\ \frac{3}{8}-\frac{x}{8} & 0<x<3\end{cases}$$

- Calculating probability values

$$P(-2 < X < -1) = \int_{-2}^{-1} f_1(x)dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

P(0.40 < X < 2.4) =

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

$$P(0.40 < X < 2.4) = \int_{0.40}^{2.4} f_2(x)dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

P(-1 < X < 1) =

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

$$P(-1 < X < 1) = \int_{-1}^{0} f_1(x)dx + \int_{0}^{1} f_2(x)dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

P(-1 < X < 7) =

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

$$P(-1 < X < 7) = \int_{-1}^{0} f_1(x)dx + \int_{0}^{3} f_2(x)dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

$$P(-1 < X < 7) = \int_{-1}^{0} f_1(x)dx + \int_{0}^{3} f_2(x)dx + \int_{3}^{7} f_3(x)dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- Calculating probability values

$$P(-1 < X < 7) = \int_{-1}^{0} f_1(x)dx + \int_{0}^{3} f_2(x)dx + \int_{3}^{7} 0 \, dx$$

Random Variables

Piecewise pdf's

A random variable X takes values between –2 and 3 with a pdf

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64} & -2 < x < 0 \\ \frac{3}{8} - \frac{x}{8} & 0 < x < 3 \end{cases}$$

- How would you calculate the cdf?

Random Variables

- Re-read sections 2.1 and 2.2
- Work as many examples & exercises as possible.