Expected Value

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72, 98, 88

Find the average if the tests are equally weighted.

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$$\frac{72 + 98 + 88}{3} = 86$$

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Find the average if the tests are equally weighted.

$$\frac{72}{3} + \frac{98}{3} + \frac{88}{3} = 86$$

Expected Value

Suppose you have the following test scores:
72, 98, 88

Find the average if the tests are equally weighted.

$$\frac{1}{3}72 + \frac{1}{3}98 + \frac{1}{3}88 = 86$$

Expected Value

Suppose you have the following test scores:
72, 98, 88

Find the average if the tests are equally weighted.

$$\frac{1}{3}72 + \frac{1}{3}98 + \frac{1}{3}88 = 86$$

What if test 1 is worth 50% of the final grade and tests 2 and 3 are 25% each?

Expected Value

Suppose you have the following test scores:
72, 98, 88

Find the average if the tests are equally weighted.

$$\frac{1}{2}72 + \frac{1}{4}98 + \frac{1}{4}88 = 82.5$$

What if test 1 is worth 50% of the final grade and tests 2 and 3 are 25% each?

Expected Value

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72, 98, 88

Find the average if the tests are equally weighted.

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Idea behind Expected Value / Mean / Average:

Multiply each value by it's appropriate weight .

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Idea behind Expected Value / Mean / Average:

Multiply each value by it's appropriate weight.

Weights must add up to 1

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Suppose you have the following test scores:

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$$\frac{1}{2}72 + \frac{1}{4}98 + \frac{1}{4}88 = 82.5$$

Idea behind Expected Value / Mean / Average:

Multiply each value by it's appropriate weight.

Weights must add up to 1 $\Rightarrow$ Analogous to probabilities

Expected Value

Expectation of a random variable

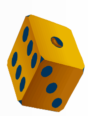
Idea:

- Multiply each possible value of the random variable *X* by it's respective probability.
- Add 'em together.

Expected Value

Example

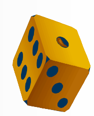
Let X = the number on a die toss.



Expected Value

Example

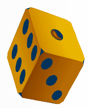
Let X = the number on a die toss. S = {1,2,3,4,5,6}



Expected Value

Example
Let X = the number on a die toss. S = {1,2,3,4,5,6}

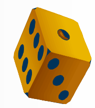
E[X] =



Expected Value

Example
Let X = the number on a die toss. S = {1,2,3,4,5,6}

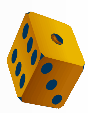
$E[X] = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \ldots + \frac{1}{6} \cdot 6$



Expected Value

Example
Let X = the number on a die toss. S = {1,2,3,4,5,6}

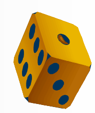
$E[X] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)$



Expected Value

Example
Let X = the number on a die toss. S = {1,2,3,4,5,6}

$E[X] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = 3.5$



Expected Value

Example
Let X = the number on a die toss. S = {1,2,3,4,5,6}

$E[X] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = 3.5$

Expected value does not have to be a possible value! (think "*averages*")

Expected Value

In general:

If the random variable *X* takes on values
x_1 with probability p_1
x_2 with probability p_2
… etc.

Then

$E[X] = x_1p_1 + x_2p_2 + \ldots + x_np_n + \ldots$

Expected Value

In general:

If the random variable *X* takes on values
x_1 with probability p_1
x_2 with probability p_2
… etc.
Then

$$\mathbf{E[X]} = \sum_i x_i p_i$$

Expected Value

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If the random variable *X* takes on values
x_1 with probability p_1
x_2 with probability p_2
… etc.
Then

$$\mathbf{E[X]} = \sum_i x_i p_i$$

Recall: $p_i = P(X = x_i) = f(x_i)$

Expected Value

In general:

If the random variable *X* takes on values
x_1 with probability p_1
x_2 with probability p_2
… etc.
Then

$$\mathbf{E[X]} = \sum_i x_i f(x_i)$$

Expected Value

Example

Toss two fair coins, and let *X* be the number of heads observed. Find the pmf and expected value for *X*.

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Expected Value

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E[X] =

Expected Value

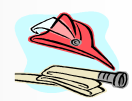
Example

Toss two fair coins, and let X be the number of heads observed. Find the pmf and expected value for X.

x	0	1	2
$f(x)$	1/4	1/2	1/4

$$E[X] = \frac{1}{4}\cdot 0 + \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 = 1$$

Expected Value



Example

In a lottery conducted to benefit the local fire company, 8000 tickets are to be sold at $5 each. The prize is $12,000. If you purchase two tickets, what is your expected net gain?

Expected Value

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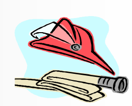
In a lottery conducted to benefit the local fire company, 8000 tickets are to be sold at $5 each. The prize is $12,000. If you purchase two tickets, what is your expected **net gain**?

x	
$f(x)$	

Expected Value

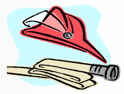


Example

In a lottery conducted to benefit the local fire company, 8000 tickets are to be sold at $5 each. The prize is $12,000. If you purchase two tickets, what is your expected net gain?

x	win	don't win
$f(x)$		

Expected Value

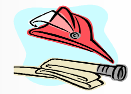


Example

In a lottery conducted to benefit the local fire company, 8000 tickets are to be sold at $5 each. The prize is $12,000. If you purchase two tickets, what is your expected net gain?

x	\$11,990	–\$10
$f(x)$		

Expected Value



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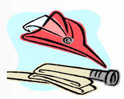
x	\$11,990	–\$10
$f(x)$	2/8000	7998/8000

Expected Value



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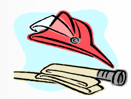
E[X] =

Expected Value



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x	\$11,990	–\$10
$f(x)$	2/8000	7998/8000

$$E[X] = \frac{2}{8000} \cdot 11990 + \frac{7998}{8000} \cdot (-10) = -\$7$$

Expected Value

✓ Discrete

$$E[X] = \sum_i x_i f(x_i)$$

Expected Value

✓ Discrete

$$E[X] = \sum_i x_i f(x_i)$$

Continuous

Expected Value

✓ Discrete

E[X] = $\sum_i x_i f(x_i)$

Continuous

E[X] = $\int_s x\, f(x)\, dx$

Expected Value

Example

For a given manufacturer, the fraction of customers that file a complaint can be modeled using the following pdf:

$$f(x) = \frac{3}{2}(1 - x^2) \qquad 0 \le x \le 1$$

Calculate the expected fraction of customers that will file a complaint.

Expected Value

Example

For a given manufacturer, the fraction of customers that file a complaint can be modeled using the following pdf:

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E[X] =

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E[X] = $\int_0^1 x\, f(x) dx =$

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$$= \frac{3}{2}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Bigg|_0^1$$

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$$= \frac{3}{2}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Bigg|_0^1$$

$$= \frac{3}{8}$$