

Money Grab-Bag

- Given 2 bags, each with 200 plastic chips
- Each chip has a dollar amount stamped on it



Money Grab-Bag

- Given 2 bags, each with 200 plastic chips
- Each chip has a dollar amount stamped on it
- Average dollar amount for each bag is \$7000.



Money Grab-Bag

- Given 2 bags, each with 200 plastic chips
- Each chip has a dollar amount stamped on it
- Average dollar amount for each bag is \$7000.

- For \$175, you are given the chance to **choose 1 chip from either bag** and will win the dollar amount on the chip.

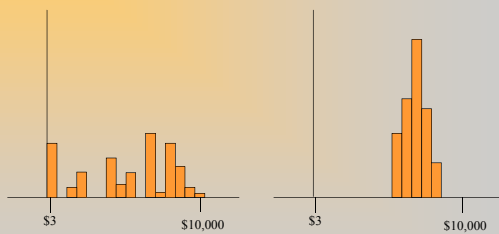


Money Grab-Bag

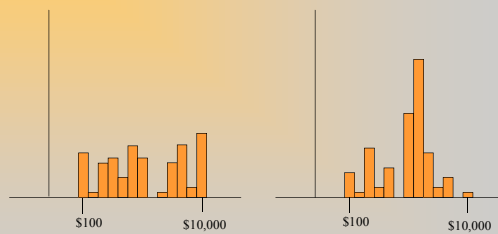
- Given 2 bags, each with 200 plastic chips
- Each chip has a dollar amount stamped on it
- Average dollar amount for each bag is \$7000.

- For \$175, you are given the chance to choose 1 chip from either bag and will win the dollar amount on the chip.

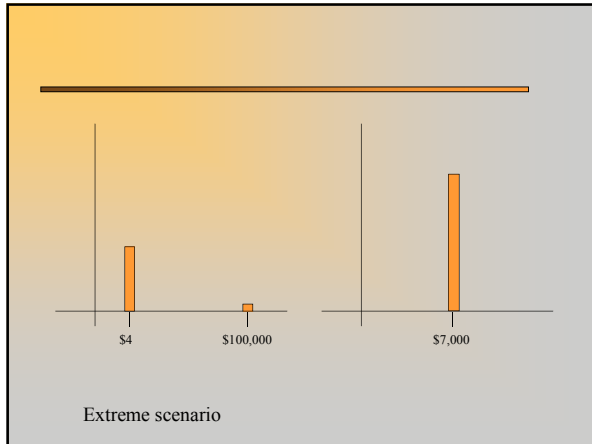
Which bag do you choose? [Does it matter?]



Same mean, very different spread.



May cover the same range of values... but still different.



Variance

Summary measures

Variance

Summary measures

Expected value : measures the central or average value of the random variable.

Variance

Summary measures

Expected value : measures the central or average value of the random variable.

not always sufficient info

Variance

Summary measures

Expected value : measures the central or average value of the random variable.

Variance

Variance

Summary measures

Expected value : measures the central or average value of the random variable.

Variance : measures the spread of the values of the random variable.

Variance

Calculating variance

- * Use the following numbers to derive an expression for variance:

8, 3, 12, 4, 5, 8, 2 (all equally likely values)

Variance

Idea:

- * What is the expected value?

$$E(X)$$

Variance

Idea:

- * What is the expected value?
- * How far away is each possible outcome from the expected value?

$$X_i - E(X)$$

Variance

Idea:

- * What is the expected value?
- * How far away is each possible outcome from the expected value?
- * Magnitude of distance

$$(X_i - E(X))^2$$

Variance

Idea:

- * What is the expected value?
- * How far away is each possible outcome from the expected value?
- * Magnitude of distance
- * Take the average of distances

$$E((X_i - E(X))^2)$$

Variance

Idea:

- * What is the expected value?
- * How far away is each possible outcome from the expected value?
- * Magnitude of distance
- * Take the average of distances

$$\text{Var}(X) = E((X_i - E(X))^2)$$

Variance

Derivation Stuff

Calculating a more user friendly expression...

Variance

Derivation Stuff

$$\text{Var}(X) = E((X_i - E(X))^2)$$

Variance

Derivation Stuff

$$\begin{aligned}\text{Var}(X) &= E((X_i - E(X))^2) \\ &= E(X_i^2 - 2X_i E(X) + (E(X))^2)\end{aligned}$$

Variance

Derivation Stuff

$$\begin{aligned}\text{Var}(X) &= E((X_i - E(X))^2) \\ &= E(X_i^2 - 2X_i E(X) + (E(X))^2) \\ &= E(X^2) - E[2X E(X)] + E[(E(X))^2]\end{aligned}$$

Variance

Derivation Stuff

$$\begin{aligned}\text{Var}(X) &= E((X_i - E(X))^2) \\ &= E(X_i^2 - 2X_i E(X) + (E(X))^2) \\ &= E(X^2) - E[2X E(X)] + E[(E(X))^2] \\ &= E(X^2) - 2E(X)E(X) + (E(X))^2\end{aligned}$$

Variance

Derivation Stuff

$$\begin{aligned}\text{Var}(X) &= E((X_i - E(X))^2) \\ &= E(X_i^2 - 2X_i E(X) + (E(X))^2) \\ &= E(X^2) - E[2X E(X)] + E[(E(X))^2] \\ &= E(X^2) - 2E(X)E(X) + (E(X))^2 \\ &= E(X^2) - 2(E(X))^2 + (E(X))^2\end{aligned}$$

Variance

$$\begin{aligned}\text{Var}(X) &= E((X_i - E(X))^2) \\ &= E(X_i^2 - 2X_i E(X) + (E(X))^2) \\ &= E(X^2) - E[2X E(X)] + E[(E(X))^2] \\ &= E(X^2) - 2E(X)E(X) + (E(X))^2 \\ &= E(X^2) - 2(E(X))^2 + (E(X))^2 \\ &= E(X^2) - (E(X))^2\end{aligned}$$

Variance

$$\begin{aligned}\text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= \sigma^2\end{aligned}$$

Variance

Definition

$$\text{Standard deviation} = \sqrt{\text{Var}(X)}$$

Variance

Definition

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\text{Var}(X)} \\ &= \sqrt{\sigma^2} \\ &= \sigma\end{aligned}$$

Variance

- Idea of standard deviation is similar to that of variance.

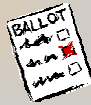
Variance

- Idea of standard deviation is similar to that of variance.
- Typically want σ to be small
(low deviation \Rightarrow accuracy)

Variance

- Idea of standard deviation is similar to that of variance.
- Typically want σ to be small
(low deviation \Rightarrow accuracy)

e.g.
– voting polls



Variance

- Idea of standard deviation is similar to that of variance.
- Typically want σ to be small
(low deviation \Rightarrow accuracy)

e.g.
– voting polls
– salary increase



Variance

- Idea of standard deviation is similar to that of variance.
- Typically want σ to be small
(low deviation \Rightarrow accuracy)

e.g.
– voting polls
– salary increase
– construction parts



Variance

- Idea of standard deviation is similar to that of variance.
- Typically want σ to be small
(low deviation \Rightarrow accuracy)

e.g.
– voting polls
– salary increase
– construction parts

- **Tolerances** are usually defined according to std. deviations

Variance

Work it out...

Recall the following pmf:

$$f(x) = \frac{4-x}{6} \quad x = 1, 2, 3$$

Calculate the **expected value**, μ , and **variance**, σ^2 , of X.

Expected Value

Recall 2.3.11

We were given a continuous random variable with the following cdf:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

- ✓ a) What is the expected value of this random variable?

Variance

Recall 2.3.11

We were given a continuous random variable with the following cdf:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

- ✓ a) What is the expected value of this random variable?
- b) What's the variance of X? standard deviation?

//

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards. Let X = the player's earnings.

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards. Let X = the player's earnings.

- 1. Calculate the pmf.

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards. Let X = the player's earnings.

- 1. Calculate the pmf.

$$\$1, \$2 \Rightarrow \text{earning} = \$3 - \$4 = -\$1$$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards. Let X = the player's earnings.

- 1. Calculate the pmf.

$$\$1, \$2 \Rightarrow \text{earning} = \$3 - \$4 = -\$1$$

$$\$1, \$3 \Rightarrow \text{earning} = \$4 - \$4 = \$0$$

...

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards. Let X = the player's earnings.

\$1, \$2	\$1, \$3	\$1, \$6	\$2, \$3	\$2, \$6	\$3, \$6
-1	0	3	1	4	5

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

\$1, \$2	\$1, \$3	\$1, \$6	\$2, \$3	\$2, \$6	\$3, \$6
-1	0	3	1	4	5
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

2. Calculate the expected value of X

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

2. Calculate the expected value of X

$$E[X] = -1 \cdot \frac{1}{6} + 0 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6}$$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

2. Calculate the expected value of X

$$E[X] = \frac{1}{6} \cdot (-1 + 0 + 3 + 1 + 4 + 5)$$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

2. Calculate the expected value of X

$$E[X] = \frac{1}{6} \cdot (-1 + 0 + 3 + 1 + 4 + 5) = \frac{1}{6} \cdot 12$$

Variance

An extension of 2.1.8

Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the cards.

1. Calculate the pmf.

$$f(x) = \frac{1}{6} \quad x = -1, 0, 3, 1, 4, 5$$

2. Calculate the expected value of X

$$E[X] = \frac{1}{6} \cdot (-1 + 0 + 3 + 1 + 4 + 5) = \frac{1}{6} \cdot 12 = \mathbf{\$2}$$